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CHEMICAL HYPERSTRUCTURES FOR STRATOSPHERIC OZONE DEPLETION

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Abstract. In this paper, we investigate the mathematical structures of chemical reactions for stratospheric ozone depletion.

1. Introduction

Until the mid-1960s, the Chapman mechanism was thought to be complete in explaining the chemical reaction of the O2 / O3 system in the stratosphere. Since then, much research has been done in this field, and as a result of calculating the steady-state concentration of ozone, we recognize there must be another natural sink that was yet undiscovered in the cause of ozone depletion in the stratosphere. It was confirmed that this secondary mechanism is a catalytic process. We want to study the interaction of chemical species for ozone depletion mathematically within natural processes, artificial processes, as well as a combination of natural and artificial processes.

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Since F. Marty[14] introduced the concept of algebraic hyperstructures, many mathematicians [8, 9] have studied them. Moreover, T. Vougiouklis[21] generalized the concept of algebraic hyperstructures and studied H_v-groups.

Even now, many researchers[1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 16, 17, 18, 19, 20] continue to study the mathematical hyperstructures of biological, chemical, and physical reactions.

Ozone destruction from environmental pollution is now underway on Earth. In this paper, we provide the mathematical hyperstructures of chemical reactions for stratospheric ozone depletion.

2. Hyperalgebraic structures

Let H be a non-empty set and a function $\cdot : H \times H \longrightarrow \wp^*(H)$ be a *hyperoperation*, where $\wp^*(H)$ is the set of all non-empty subset of H. The couple $(H, \cdot)$ is called a *hypergroupoid*. For the subset A, B of H, we define $A \cdot B = \cup_{a \in A, b \in B} a \cdot b$, and for a singleton $\{a\}$ we denote $\{a\} \cdot B = a \cdot B$ and $B \cdot \{a\} = B \cdot a$.

Definition 2.1. The hypergroupoid $(H, \cdot)$ is called a *semihypergroup* if

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z, \text{ for all } x, y, z \in H.$$

In this case, the hyperoperation $(\cdot)$ is called *associate*.

The hypergroupoid $(H, \cdot)$ is called an H_v*-semigroup* if

$$x \cdot (y \cdot z) \cap (x \cdot y) \cdot z \neq \emptyset, \text{ for all } x, y, z \in H.$$

In this case, the hyperoperation $(\cdot)$ is called *weak associate*.

The hypergroupoid $(H, \cdot)$ is called a *quasihypergroup* if

$$x \cdot H = H \cdot x = H, \text{ for all } x \in H.$$

The hyperoperation $(\cdot)$ is called *commutative* if

$$x \cdot y = y \cdot x, \text{ for all } x, y \in H.$$

The hypergroupoid $(H, \cdot)$ is called a *hypergroup* if it is a semihypergroup and a quasihypergroup. If a non-empty subset B of H is a hypergroup, then $(B, \cdot)$ is called a *subhypergroup* of H.

The hypergroupoid $(H, \cdot)$ is called an H_v*-group* if it is an H_v-semigroup and a quasihypergroup. If a non-empty subset B of H is an H_v-group, then $(B, \cdot)$ is called a H_v*-subgroup* of H.

The hypergroupoid $(H, \cdot)$ is called a *commutative hypergroup* if it is a hypergroup with a commutative hyperoperation $(\cdot)$.

The hypergroupoid $(H, \cdot)$ is called a *commutative* H_v*-group* if it is an H_v-group with a commutative hyperoperation $(\cdot)$.

Let $(H_1, \cdot)$ and $(H_2, *)$ be two H_v-groups. A map $f : H_1 \longrightarrow H_2$ is called an H_v*-homomorphism* or *weak homomorphism* if

$$f(x \cdot y) \cap f(x) * f(y) \neq \emptyset, \text{ for all } x, y \in H_1.$$

f is called an *inclusion homomorphism* if

$$f(x \cdot y) \subset f(x) * f(y), \text{ for all } x, y \in H_1.$$

Finally, f is called a *strong homomorphism* if

$$f(x \cdot y) = f(x) * f(y), \text{ for all } x, y \in H_1.$$

If f is onto, one to one and strong homomorphism, then it is called an *isomorphism*. In this case, H_1 and H_2 are called *isomorphic* and we write $H_1 \cong H_2$.

3. Chemical reactions in stratospheric ozone depletion

The depletion of stratospheric ozone has caused important changes in tropospheric climate and will continue to do so for decades to come.

In this paper, we study some algebraic hyperstructures for stratospheric ozone depletion by a catalyst.

Today it is well established that stratospheric ozone is produced by the photolysis of molecular oxygen (O_2) at ultraviolet (hv) wavelengths below $242nm$ in the chemical reaction ($R1$). Müller introduced the following ozone depletion steps([7], 1.1.2) and we can find in [15] :

$$\begin{array}{lrcl}
(R1) & O_2 + hv & \longrightarrow & 2O \\
(R2) & O + O_2 + M & \longrightarrow & O_3 + M \\
(R3) & O_3 + hv & \longrightarrow & O + O_2 \\
(R4) & O + O_3 & \longrightarrow & 2O_2 \\
(R5) & XO + O & \longrightarrow & X + O_2 \\
(R6) & X + O_3 & \longrightarrow & XO + O_2
\end{array}$$

where hv denotes an ultraviolet photon, M denotes a collision partner (N_2 or O_2, etc.) that is not affected by the reaction, and X is a catalyst (Cl, NO, H, OH, etc.).

We studied the ozone destructive reaction in three directions : ozone production and destruction in natural processes (R1) to (R4), artificial ozone depletion reactions (R5) to (R6), and reactions involving both natural and artificial processes (R1) to (R6).

In addition, the mathematical interaction ($a \oplus_{i1} b = a \oplus_{i1} b, i = 1, 2, 3$: refer definitions 3.1, 3.3, 3.5) and the chemical interaction ($a \oplus_{i2} b = a \oplus_{i1} a \cup a \oplus_{i1} b \cup b \oplus_{i1} b, i = 1, 2, 3$: refer definitions 3.2, 3.4, 3.6) occurring in reaction of each field were investigated.

We give a hyperoperation table for the set $\{O, O_2, O_3, X, XO\}$ of the chemical elements obtained from the above chemical reactions.

Definition 3.1. Let G be the set $\{O, O_2, O_3\}$ of the chemical elements and a *hyperoperation* $\oplus_{11}$ on G is defined as follows:

$$\oplus_{11} : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ is the set of all non-empty subset of G. For all $x, y \in G$, $x \oplus_{11} y$ is defined as the union of all the possible chemical reactions which appear in the above primary reactions $(R1) \sim (R4)$. If $x \oplus_{11} y$ does not appear in the above reactions $(R1) \sim (R4)$, then we define as follows:

$$(R0) \qquad x \oplus_{11} y = \{x, y\}$$

Then we can define $x \oplus_{11} y$ for elements $x, y \in \{O, O_2, O_3\}$ as follows.

First for the $x = O$,

$$\begin{aligned}
O \oplus_{11} O &= O \oplus_{11} O &&= \{O\} \text{ by } (R0). \\
O \oplus_{11} O_2 &= O_2 \oplus_{11} O &&= \{O_3\} \text{ by } (R2). \\
O \oplus_{11} O_3 &= O_3 \oplus_{11} O &&= \{O_2\} \text{ by } (R4).
\end{aligned}$$

Next for the $x = O_2$,

$$O_2 \oplus_{11} O_2 \quad = O_2 \oplus_{11} O_2 \quad = \{O\} \text{ by } (R1).$$

$$O_2 \oplus_{11} O_3 \quad = O_3 \oplus_{11} O_2 \quad = \{O_2, O_3\} \text{ by } (R0).$$

For the $x = O_3$,

$$O_3 \oplus_{11} O_3 \quad = O_3 \oplus_{11} O_3 \quad = \{O, O_2\} \text{ by } (R3).$$

Then we have a hyperoperation table for the set $\{O, O_2, O_3\}$.

$\oplus_{11}$	O	O_2	O_3
O	O	O_3	O_2
O_2	O_3	O	O_2, O_3
O_3	O_2	O_2, O_3	O, O_2

Table 1. Hyperoperation table for the operation $\oplus_{11}$

In the above table, if we change the name from O, O_2 and O_3 to a, b and c, respectively, then we have the following Table 2:

$\oplus_{11}$	a	b	c
a	$\{a\}$	$\{c\}$	$\{b\}$
b	$\{c\}$	$\{a\}$	$\{b,c\}$
c	$\{b\}$	$\{b,c\}$	$\{a,b\}$

Table 2. Hyperoperation table for the operation $\oplus_{11}$

The operation $\oplus_{11}$ is the definition of the reaction for each chemical element, but the following operation $\oplus_{12}$ is the definition of the reaction in the chemical element set.

DEFINITION 3.2. Let G be the set $\{O, O_2, O_3\}$ of the chemical elements and a *hyperoperation* $\oplus_{12}$ on G is defined as follows:

$$\oplus_{12} : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ is the set of all non-empty subset of G. For all $x, y \in G$, $x \oplus_{12} y$ is defined as follows:

$$(x \oplus_{11} x) \cup (x \oplus_{11} y) \cup (y \oplus_{11} y)$$

For example, in the case $O_2 \oplus_{12} O_3$,

$$\begin{aligned}
&O_2 \oplus_{12} O_3 \\
&= (O_2 \oplus_{11} O_2) \cup (O_2 \oplus_{11} O_3) \cup (O_3 \oplus_{11} O_3) \\
&= \{O\} \cup \{O_2, O_3\} \cup \{O, O_2\} \\
&= \{O, O_2, O_3\}
\end{aligned}$$

Then we can define $x \oplus_{12} y$ for elements $x, y \in \{O, O_2, O_3\}$ as follows.

First for the $x = O$,

$$\begin{aligned}
O \oplus_{12} O \quad &= O \oplus_{12} O \quad &&= \{O\}. \\
O \oplus_{12} O_2 \quad &= O_2 \oplus_{12} O \quad &&= \{O, O_3\}. \\
O \oplus_{12} O_3 \quad &= O_3 \oplus_{12} O \quad &&= \{O, O_2\}
\end{aligned}$$

Next for the $x = O_2$,

$$\begin{aligned}
O_2 \oplus_{12} O_2 \quad &= O_2 \oplus_{12} O_2 \quad &&= \{O\}. \\
O_2 \oplus_{12} O_3 \quad &= O_3 \oplus_{12} O_2 \quad &&= \{O, O_2, O_3\}.
\end{aligned}$$

For the $x = O_3$,

$$O_3 \oplus_{12} O_3 \quad = O_3 \oplus_{12} O_3 \quad = \{O, O_2\}.$$

If we change the name from O, O_2 and O_3 to a, b and c, respectively, then we have the following Table 3:

$\oplus_{12}$	a	b	c
a	$\{a\}$	$\{a,c\}$	$\{a,b\}$
b	$\{a,c\}$	$\{a\}$	$\{a,b,c\}$
c	$\{a,b\}$	$\{a,b,c\}$	$\{a,b\}$

TABLE 3. Hyperoperation table for the operation $\oplus_{12}$

DEFINITION 3.3. Let G be the set $\{O, O_2, O_3, X, XO\}$ of the chemical elements. Using chemical reactions $(R5)$ and $(R6)$ instead of chemical reactions $(R1) \sim (R4)$ in Definition 3.1, let's define a *hyperoperation* $\oplus_{21}$ on G in the same way as Definition 3.1.

And let's change the name from O, O_2, O_3, X and XO to a, b, c, d and e respectively, then we have the following Table 4:

$\oplus_{21}$	a	b	c	d	e
a	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$	$\{b,d\}$
b	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{b,e\}$	$\{c,e\}$
d	$\{a,d\}$	$\{b,d\}$	$\{b,e\}$	$\{d\}$	$\{d,e\}$
e	$\{b,d\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

TABLE 4. Hyperoperation table for the operation $\oplus_{21}$

DEFINITION 3.4. Let G be the set $\{O, O_2, O_3, X, XO\}$ of the chemical elements. Using chemical reactions $(R5)$ and $(R6)$ instead of chemical reactions $(R1) \sim (R4)$ in Definition 3.2, let's define a *hyperoperation* $\oplus_{22}$ on G in the same way as Definition 3.2.

And let's change the name from O, O_2, O_3, X and XO to a, b, c, d and e respectively, then we have the following Table 5:

Define the third hyperoperations $\oplus_{31}$ and $\oplus_{32}$.

$\oplus_{22}$	a	b	c	d	e
a	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$	$\{a,b,d,e\}$
b	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{b,c,d,e\}$	$\{c,e\}$
d	$\{a,d\}$	$\{b,d\}$	$\{b,c,d,e\}$	$\{d\}$	$\{d,e\}$
e	$\{a,b,d,e\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

TABLE 5. Hyperoperation table for the operation $\oplus_{22}$

DEFINITION 3.5. Let G be the set $\{O, O_2, O_3, X, XO\}$ of the chemical elements. Using chemical reactions $(R1) \sim (R6)$ instead of chemical reactions $(R1) \sim (R4)$ in Definition 3.1, let's define a *hyperoperation* $\oplus_{31}$ on G in the same way as Definition 3.1.

And let's change the name from O, O_2, O_3, X and XO to a, b, c, d and e respectively, then we have the following Table 6:

$\oplus_{31}$	a	b	c	d	e
a	$\{a\}$	$\{c\}$	$\{b\}$	$\{a,d\}$	$\{b,d\}$
b	$\{c\}$	$\{a\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{b,e\}$	$\{c,e\}$
d	$\{a,d\}$	$\{b,d\}$	$\{b,e\}$	$\{d\}$	$\{d,e\}$
e	$\{b,d\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

TABLE 6. Hyperoperation table for the operation $\oplus_{31}$

DEFINITION 3.6. Let G be the set $\{O, O_2, O_3, X, XO\}$ of the chemical elements. Using chemical reactions $(R1) \sim (R6)$ instead of chemical reactions $(R1) \sim (R4)$ in Definition 3.2, let's define a *hyperoperation* $\oplus_{32}$ on G in the same way as Definition 3.2.

And let's change the name from O, O_2, O_3, X and XO to a, b, c, d and e respectively, then we have the following Table 7:

$\oplus_{32}$	a	b	c	d	e
a	$\{a\}$	$\{a,c\}$	$\{a,b\}$	$\{a,d\}$	$\{a,b,d,e\}$
b	$\{a,c\}$	$\{a\}$	$\{a,b,c\}$	$\{a,b,d\}$	$\{a,b,e\}$
c	$\{a,b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,c,e\}$
d	$\{a,d\}$	$\{a,b,d\}$	$\{a,b,d,e\}$	$\{d\}$	$\{d,e\}$
e	$\{a,b,d,e\}$	$\{a,b,e\}$	$\{a,b,c,e\}$	$\{d,e\}$	$\{e\}$

TABLE 7. Hyperoperation table for the operation $\oplus_{32}$

THEOREM 3.7. *Let $G = \{a, b, c\}$ be the set of the chemical elements obtained from the chemical reactions $(R1) \sim (R4)$. Let $(G, \oplus_{11})$ and $(G, \oplus_{12})$ be the hypergroupoids.*

Then we have the following.

(1) *The hypergroupoid $(G, \oplus_{11})$ is a commutative quasihypergroup but not an H_v-semigroup.*
(2) *The hypergroupoid $(G, \oplus_{12})$ is a commutative H_v-group but not a hypergroup.*

Proof. (1) Since $x \oplus_{11} G = G \oplus_{11} x = G$ for all $x \in G$, $(G, \oplus_{11})$ is a quasihypergroup. Clearly $\oplus_{11}$ is commutative. But since

$$a \oplus_{11} (a \oplus_{11} b) = \{b\} \text{ and } (a \oplus_{11} a) \oplus_{11} b = \{c\},$$

the hyperoperation $\oplus_{11}$ is not weak associate. Hence it is not an H_v-semigroup.

(2) In the case $(G, \oplus_{12})$, since $x \oplus_{12} G = G \oplus_{12} x = G$ for all $x \in G$, $(G, \oplus_{12})$ is a quasihypergroup. Clearly $\oplus_{12}$ is commutative and we have the following: for $x, y, z \in G$

$$x \oplus_{12} (y \oplus_{12} z) \cap (x \oplus_{12} y) \oplus_{12} z \ni a$$

Thus the hypergroupoid $(G, \oplus_{12})$ is a commutative H_v-group. But the hyperoperation $\oplus_{12}$ is not associative; for example, since $a \oplus_{12} (a \oplus_{12} b) = \{a, b\}$ and $(a \oplus_{12} a) \oplus_{12} b = \{a, c\}$, we have

$$a \oplus_{12} (a \oplus_{12} b) \neq (a \oplus_{12} a) \oplus_{12} b.$$

Hence $(G, \oplus_{12})$ is not a hypergroup. □

THEOREM 3.8. *Let $G = \{a, b, c, d, e\}$ be the set of the chemical elements obtained from the chemical reactions $(R5) \sim (R6)$. Let $(G, \oplus_{21})$ and $(G, \oplus_{22})$ be the hypergroupoids.*

Then we have the following.

(1) *The hyperstructure $(G, \oplus_{21})$ is a commutative H_v-group.*

(2) *The hyperstructure $(G, \oplus_{22})$ is a commutative hypergroup.*

(3) *For $i = 1$ or 2, the hyperstructures $(\{a, b\}, \oplus_{2i})$, $(\{a, c\}, \oplus_{2i})$, $(\{a, d\}, \oplus_{2i})$, $(\{b, c\}, \oplus_{2i})$, $(\{b, d\}, \oplus_{2i})$, $(\{b, e\}, \oplus_{2i})$, $(\{c, e\}, \oplus_{2i})$, $(\{d, e\}, \oplus_{2i})$ are commutative subhypergroups of $(G, \oplus_{2i})$ and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_{2i}$	x	y
x	$\{x\}$	$\{x,y\}$
y	$\{x,y\}$	$\{y\}$

(4) *For $i = 1$ or 2, the hyperstructures $(\{a, b, c\}, \oplus_{2i})$, $(\{a, b, d\}, \oplus_{2i})$, $(\{b, c, e\}, \oplus_{2i})$ and $(\{b, d, e\}, \oplus_{2i})$ are commutative subhypergroups of $(G, \oplus_{2i})$ and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_{2i}$	x	y	z
x	$\{x\}$	$\{x,y\}$	$\{x,z\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$

(5) *The hyperstructures* $(\{a,b,d,e\},\oplus_{21})$ *and* $(\{b,c,d,e\},\oplus_{21})$ *are commutative* H_v*-subsemigroups of* $(G,\oplus_{21})$ *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_{21}$	x	y	z	w
x	$\{x\}$	$\{x,y\}$	$\{x,z\}$	$\{y,z\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$	$\{y,w\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$	$\{z,w\}$
w	$\{y,z\}$	$\{y,w\}$	$\{z,w\}$	$\{w\}$

The hyperstructures $(\{a,b,d,e\},\oplus_{22})$ *and* $(\{b,c,d,e\},\oplus_{22})$ *are commutative subhypergroups of* $(G,\oplus_{22})$ *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_{22}$	x	y	z	w
x	$\{x\}$	$\{x,y\}$	$\{x,z\}$	$\{x,y,z,w\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$	$\{y,w\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$	$\{z,w\}$
w	$\{x,y,z,w\}$	$\{y,w\}$	$\{z,w\}$	$\{w\}$

Proof. (1) Since $a \oplus_{21} G = G \oplus_{21} a = \{a, b, c, d\}$, $(G, \oplus_{21})$ is not a quasihypergroup. Clearly $\oplus_{21}$ is commutative and we have the following: for $x, y, z \in G$

$$\begin{cases} x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni a, & \text{if } a \in \{x,y,z\} \text{ and } e \notin \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni b, & \text{if } a \in \{x,y,z\} \text{ and } e \in \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni b, & \text{if } b \in \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni c, & \text{if } c \in \{x,y,z\} \text{ and } d \notin \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni b, & \text{if } c \in \{x,y,z\} \text{ and } d \in \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni d, & \text{if } d \in \{x,y,z\} \text{ and } c \notin \{x,y,z\}; \\ x \oplus_{21} (y \oplus_{21} z) \cap (x \oplus_{21} y) \oplus_{21} z \ni e, & \text{if } e \in \{x,y,z\} \text{ and } a \notin \{x,y,z\}; \end{cases}$$

Thus the hypergroupoid $(G, \oplus_{21})$ is a commutative H_v-semigroup. However, the hyperoperation $\oplus_{21}$ is not associative; for example, since $a \oplus_{21} (a \oplus_{21} e) = \{a, b, d\}$ and $(a \oplus_{21} a) \oplus_{21} e = \{b, d\}$, we have

$$a \oplus_{21} (a \oplus_{21} e) \neq (a \oplus_{21} a) \oplus_{21} e.$$

Hence $(G, \oplus_{21})$ is not a hypergroup.

(2) Since $x \oplus_{22} G = G \oplus_{22} x = G$ for all $x \in G$, $(G, \oplus_{22})$ is a quasihypergroup.

For $x, y, z \in G$, in case $\{x, y, z\} = \{\alpha\}$ for some element $\alpha \in \{x, y, z\}$

$$x \oplus_{22} (y \oplus_{22} z) = \{\alpha\} = (x \oplus_{22} y) \oplus_{22} z.$$

In case $\{x, y, z\} = \{\alpha, \beta\}$ for the different elements $\alpha, \beta \in \{x, y, z\}$

$$\begin{cases} x \oplus_{22} (y \oplus_{22} z) = \{a, b, d, e\} = (x \oplus_{22} y) \oplus_{22} z, & \text{if } \alpha = a \text{ and } \beta = e; \\ x \oplus_{22} (y \oplus_{22} z) = \{b, c, d, e\} = (x \oplus_{22} y) \oplus_{22} z, & \text{if } \alpha = c \text{ and } \beta = d; \\ x \oplus_{22} (y \oplus_{22} z) = \{\alpha, \beta\} = (x \oplus_{22} y) \oplus_{22} z, & \text{if otherwise.} \end{cases}$$

Finally, for the different elements x, y, z we have the following:

$$\begin{cases} x \oplus_{22} (y \oplus_{22} z) = \{a, b, c, d, e\} = (x \oplus_{22} y) \oplus_{22} z, \\ \qquad\qquad\qquad\qquad \text{if } \{x,y,z\} = \{a,c,d\} \text{ or } \{a,c,e\}; \\ x \oplus_{22} (y \oplus_{22} z) = \{a, b, d, e\} = (x \oplus_{22} y) \oplus_{22} z, \\ \qquad\qquad\qquad\qquad \text{if } \{x,y,z\} = \{a,b,e\} \text{ or } \{a,d,e\}; \\ x \oplus_{22} (y \oplus_{22} z) = \{b, c, d, e\} = (x \oplus_{22} y) \oplus_{22} z, \\ \qquad\qquad\qquad\qquad \text{if } \{x,y,z\} = \{b,c,d\} \text{ or } \{c,d,e\}; \\ x \oplus_{22} (y \oplus_{22} z) = \{x, y, z\} = (x \oplus_{22} y) \oplus_{22} z, \\ \qquad\qquad\qquad\qquad \text{if otherwise.} \end{cases}$$

Thus the hypergroupoid $(G, \oplus_{22})$ is a commutative hypergroup.

(3) It is obvious.

(4) Clearly all hyperstructures are quasihypergroups.

For $x, y, z \in (\{x, y, z\}, \oplus_{2i})$, in case $\{x, y, z\} = \{\alpha\}$ for some element $\alpha \in \{x, y, z\}$

$$x \oplus_{2i} (y \oplus_{2i} z) = \{\alpha\} = (x \oplus_{2i} y) \oplus_{2i} z.$$

In case $\{x, y, z\} = \{\alpha, \beta\}$ for the different elements $\alpha, \beta \in \{x, y, z\}$

$$x \oplus_{2i} (y \oplus_{2i} z) = \{\alpha, \beta\} = (x \oplus_{2i} y) \oplus_{2i} z.$$

Finally, for the different elements x, y, z we have the following:

$$x \oplus_{2i} (y \oplus_{2i} z) = \{x, y, z\} = (x \oplus_{2i} y) \oplus_{2i} z.$$

Therefore $(\{x, y, z\}, \oplus_{2i})$ is a commutative subhypergroup of $(G, \oplus_{2i})$. The hyperstructures are obviously isomorphic.

(5) Let's define a map $f : (\{a, b, d, e\}, \oplus_{21}) \longrightarrow (\{b, c, d, e\}, \oplus_{21})$

$$f(a) = c, \quad f(b) = b, \quad f(d) = e, \quad f(e) = d.$$

Then we can easily show that f is an isomorphism.

Since $x \oplus_{21} \{x, y, z, w\} = \{x, y, z, w\} \oplus_{21} x = \{x, y, z\}$, $(\{x, y, z, w\}, \oplus_{21})$ is not a quasihypergroup. For $x', y', z' \in (\{x, y, z, w\}, \oplus_{21})$, in case $\{x, w\} \subset \{x', y', z'\}$,

$$\{y, z\} \subset x' \oplus_{21} (y' \oplus_{2i} z') \cap (x' \oplus_{2i} y') \oplus_{21} z'.$$

In case $\{x, w\} \not\subset \{x', y', z'\}$ and $\alpha \in \{x', y', z'\}$,

$$\alpha \in x' \oplus_{21} (y' \oplus_{21} z') \cap (x' \oplus_{21} y') \oplus_{21} z'.$$

Thus the hyperstructure $(\{x, y, z, w\}, \oplus_{21})$ is a commutative H_v-subsemigroup of $(G, \oplus_{21})$, where it is not associative; for example, since $x \oplus_{21} (y \oplus_{21} w) = \{x, y, z\}$ and $(x \oplus_{21} y) \oplus_{21} w = \{y, z, w\}$, we have

$$x \oplus_{21} (y \oplus_{21} w) \neq (x \oplus_{21} y) \oplus_{21} w.$$

Hence $(\{x, y, z, w\}, \oplus_{21})$ is not a semihypergroup.

Next let's define a map $f : (\{a, b, d, e\}, \oplus_{22}) \longrightarrow (\{b, c, d, e\}, \oplus_{22})$

$$f(a) = c, \quad f(b) = b, \quad f(d) = e, \quad f(e) = d.$$

Then we can easily show that f is an isomorphism.

Since $x \oplus_{22} \{x, y, z, w\} = \{x, y, z, w\} \oplus_{22} x = \{x, y, z, w\}$, $(\{x, y, z, w\}, \oplus_{22})$ is a quasihypergroup. For $x', y', z' \in (\{x, y, z, w\}, \oplus_{22})$, in case $\{x, w\} \subset \{x', y', z'\}$,

$$x' \oplus_{22} (y' \oplus_{2i} z') = \{x, y, z, w\} = (x' \oplus_{2i} y') \oplus_{22} z'.$$

In case $\{x, w\} \not\subset \{x', y', z'\}$ we have the following:

$$\begin{cases} x' \oplus_{22} (y' \oplus_{22} z') = \{\alpha\} = (x' \oplus_{22} y') \oplus_{22} z', & \text{if } \{x', y', z'\} = \{\alpha\}; \\ x' \oplus_{22} (y' \oplus_{22} z') = \{\alpha, \beta\} = (x' \oplus_{22} y') \oplus_{22} z', & \text{if } \{x', y', z'\} = \{\alpha, \beta\}; \\ x' \oplus_{22} (y' \oplus_{22} z') = \{x', y', z'\} = (x' \oplus_{22} y') \oplus_{22} z', & \text{if otherwise.} \end{cases}$$

Thus the hyperstructure $(\{x, y, z, w\}, \oplus_{22})$ is a commutative subhypergroup of $(G, \oplus_{22})$. □

THEOREM 3.9. *Let $G = \{a, b, c, d, e\}$ be the set of the chemical elements obtained from the chemical reactions $(R1) \sim (R6)$. Let $(G, \oplus_{31})$ and $(G, \oplus_{32})$ be the hypergroupoids.*

Then we have the following.

(1) *The hypergroupoid* $(G, \oplus_{31})$ *is a commutative but not a quasihypergroup and an* H_v*-semigroup.*

(2) *The hypergroupoid* $(G, \oplus_{32})$ *is a commutative* H_v*-group but not a hypergroup.*

(3) *For* $i = 1$ *or* 2*, the hyperstructures* $(\{a, d\}, \oplus_{3i})$ *and* $(\{d, e\}, \oplus_{3i})$ *are commutative subhypergroups of* $(G, \oplus_{3i})$ *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_{3i}$	x	y
x	$\{x\}$	$\{x,y\}$
y	$\{x,y\}$	$\{y\}$

(4) *The hyperstructures* $(\{a, b, c\}, \oplus_{31})$ *is commutative sub-quasihypergroup of* $(G, \oplus_{31})$ *but not* H_v*-subsemigroup.*

$\oplus_{31}$	a	b	c
a	$\{a\}$	$\{c\}$	$\{b\}$
b	$\{c\}$	$\{a\}$	$\{b,c\}$
c	$\{b\}$	$\{b,c\}$	$\{a,b\}$

The hyperstructures $(\{a, b, c\}, \oplus_{32})$ *is commutative* H_v*-subgroups of* $(G, \oplus_{32})$.

$\oplus_{32}$	a	b	c
a	$\{a\}$	$\{a,c\}$	$\{a,b\}$
b	$\{a,c\}$	$\{a\}$	$\{a,b,c\}$
c	$\{a,b\}$	$\{a,b,c\}$	$\{a,b\}$

Proof. (1) Since $a \oplus_{31} G = G \oplus_{31} a = \{a, b, c, d\}$, $(G, \oplus_{31})$ is not a quasihypergroup. Clearly $\oplus_{31}$ is commutative. But since

$$a \oplus_{31} (a \oplus_{31} b) = \{b\} \text{ and } (a \oplus_{31} a) \oplus_{31} b = \{c\},$$

the hyperoperation $\oplus_{31}$ is not weak associate. Hence it is not an H_v-semigroup.

(2) In the case $(G, \oplus_{32})$, since $x \oplus_{32} G = G \oplus_{32} x = G$ for all $x \in G$, $(G, \oplus_{32})$ is a quasihypergroup. Clearly $\oplus_{32}$ is commutative and we have the following: for $x, y, z \in G$

$$\begin{cases} x \oplus_{32} (y \oplus_{32} z) \cap (x \oplus_{32} y) \oplus_{32} z \ni d \text{ or } e, & \text{if } \{x, y, z\} \subset \{d, e\}; \\ x \oplus_{32} (y \oplus_{32} z) \cap (x \oplus_{32} y) \oplus_{32} z \ni a, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(G, \oplus_{32})$ is a commutative H_v-group. However, the hyperoperation $\oplus_{32}$ is not associative; for example, since $a \oplus_{32} (a \oplus_{32} b) = \{a, b\}$ and $(a \oplus_{32} a) \oplus_{32} b = \{a, c\}$, we have

$$a \oplus_{32} (a \oplus_{32} b) \neq (a \oplus_{32} a) \oplus_{32} b.$$

Hence $(G, \oplus_{32})$ is not a hypergroup.

(3) It is clear.

(4) Since $x \oplus_{31} \{a, b, c\} = \{a, b, c\} \oplus_{31} x = \{a, b, c\}$ for all $x \in \{a, b, c\}$, $(\{a, b, c\}, \oplus_{31})$ is a sub-quasihypergroup of $(G, \oplus_{31})$. Clearly $\oplus_{31}$ is commutative. But since

$$a \oplus_{31} (a \oplus_{31} b) = \{b\} \text{ and } (a \oplus_{31} a) \oplus_{31} b = \{c\},$$

the hyperoperation $\oplus_{31}$ is not weak associate. Hence it is not an H_v-subsemigroup.

In the case $(\{a,b,c\},\oplus_{32})$, since $x\oplus_{32}\{a,b,c\}=\{a,b,c\}\oplus_{32}x=\{a,b,c\}$ for all $x\in\{a,b,c\}$, $(\{a,b,c\},\oplus_{32})$ is a quasihypergroup. Clearly $\oplus_{32}$ is commutative and we have the following: for $x,y,z\in\{a,b,c\}$

$$a\in x\oplus_{32}(y\oplus_{32}z)\cap(x\oplus_{32}y)\oplus_{32}z.$$

Thus the hypergroupoid $(\{a,b,c\},\oplus_{32})$ is a commutative H_v-subgroup of $(G,\oplus_{32})$. However, the hyperoperation $\oplus_{32}$ is not associative; for example, since $a\oplus_{32}(a\oplus_{32}b)=\{a,b\}$ and $(a\oplus_{32}a)\oplus_{32}b=\{a,c\}$, we have

$$a\oplus_{32}(a\oplus_{32}b)\neq(a\oplus_{32}a)\oplus_{32}b.$$

Hence $(\{a,b,c\},\oplus_{32})$ is not a subhypergroup of $(G,\oplus_{32})$. □

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